Simulated Annealing With Particle Swarm Optimization For Optimal Design Of Truss Structures With Multiple Frequency Constraints

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ABSTRACT
In this work, the hybrid algorithm called SAwPSO (Simulated Annealing with Particle Swarm Optimization) is used for optimal design of truss structures with multiple frequency constraints. These kinds of problems are difficult to optimize because they are multimodal and have small feasible regions. The SAwPSO works as follows: SA selects the PSO parameters and then PSO optimizes the problem. Three benchmark truss optimization problems with multiple frequency constraints are explored for the validity of the SAwPSO algorithm. Numerical results indicate that SAwPSO can minimize the overall weight of truss structures subjected to multiple frequency constraints.

Keywords: Frequency constraints; optimal design; Simulated Annealing, particle Swarm Oprimization.

1. INTRODUCTION

Size optimization is a fundamental type of truss optimization, where the ultimate goal is to obtain the best bar sections. Optimizing the weight of structures with frequency constraints can be considered a difficult problem to solve, due to the fact that weight reduction conflicts with frequency restrictions (Talbi, 2009) .The first to solve this problem was Bellagamba & Yang (1981) and, since then, several researchers have been introducing different optimization algorithms, but this area of research has not been fully investigated so far.

The most important works that involve optimization methods (mathematical programming and optimization metaheuristics) to solve this type of problem are: Lin et al. (1982) with Bi-factor algorithm, Grandhi e Venkayya (1988) with Optimality Criterion (OC), Gomes (2011) used Particle Swarm Optimization (PSO), Kaveh e Zolghadr (2011) with Charged System Search (CSS), Miguel & Fadel Miguel (2012) employed Harmony Search (HS), Gonçalves et al. (2015) with Search group algorithm (SGA), Kaveh e Ilchi Ghazaan (2017) used Vibrating Particles System (VPS), Ho-Huu et al. (2018) with Novel Differential Evolution (ReDE), among others.

In this work the hybrid algorithm SAwPSO is used for size optimization problems of truss structures with Multiple Frequency Constraints. The validity of SAwPSO is confirmed by testing for three size optimization problems of truss structures. The remainder of this article is structured as follows. Section 2 describes the mathematical formulation of truss optimization. The SAwPSO is briefly presented in

Sect. 3. Section 4 presents three benchmark numerical examples to illustrate the efficiency of the SAwPSO. Finally, in Sect. 5, conclusions are presented.

2. PROBLEM DEFINITION

The objective of the structural optimization problem is to minimize the weight of the truss, achieving the optimal cross-sectional areas of the elements, satisfying some constraints of natural frequencies. The mathematical formulation for this problem can be expressed by

$$\begin{aligned}
&\text{Find,} \quad X = \{A\}, \text{ where} \quad A = \{A_1, A_2, \ldots, A_n\} \\
&\text{Minimize} \quad W(X) = \sum_{i=1}^{n} \rho_i A_i L_i \\
&\text{Subject to} \quad \begin{cases} f_q - f_q^{min} \geq 0 \\ f_r - f_r^{max} \leq 0 \\ A_i^{min} \leq A_i \leq A_i^{max} \end{cases}
\end{aligned} \tag{1}$$

where W(X) is the total weight of the minimized truss; n is the total number of structure members; ρ_i, A_i and L_i represent the density of the material, the cross-sectional area and the length of member i, respectively; fq and fr are the natural frequencies of the structure, respectively, and the subscripts "max" and "min" denote the maximum and minimum permitted limits, respectively.

3. SAwPSO ALGORITHM

The basis of the hybrid algorithm is very simple. Each algorithm is working separately, each one evaluating a different function. In the SAwPSO, the SA selects the initial parameters of the PSO (c_1, c_2, N, α), and the PSO is evaluating the objective function (weight optimization of truss structures). In this way, both algorithms perform the optimization work until it meets the established stopping criterion. Figure 1 shows the SAwPSO flowchart.

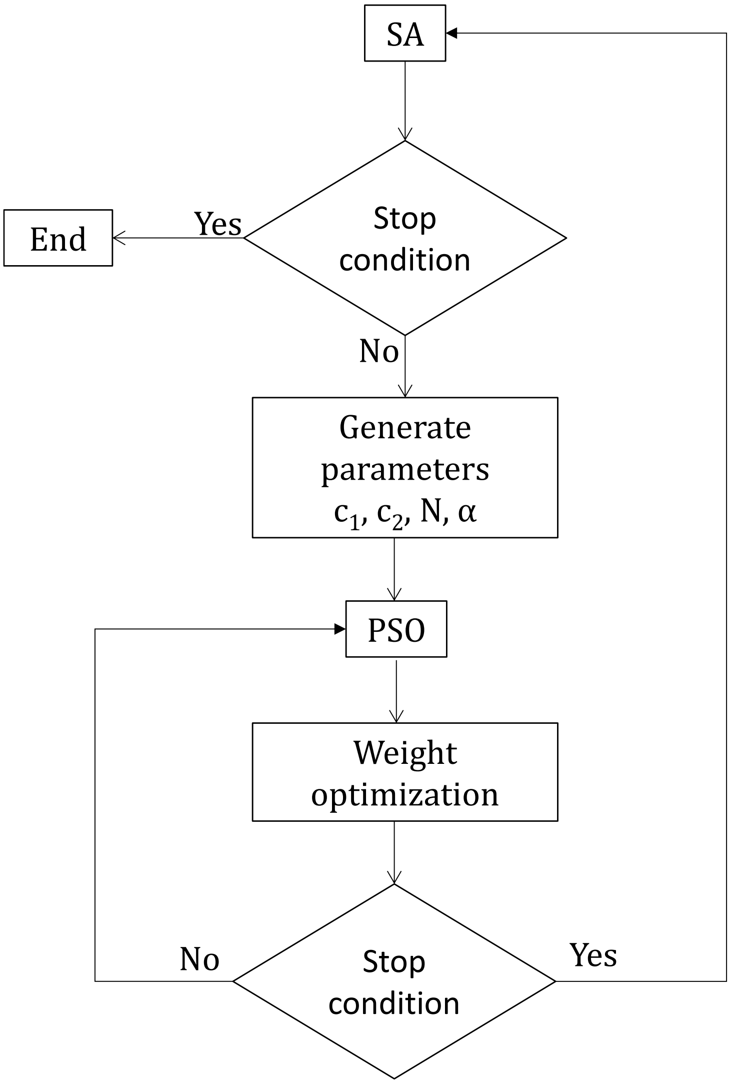

Figure 1. The SAwPSO flowchart.

4. TRUSS PROBLEMS AND DISCUSSIONS

Three trusses with 72 members, 120 and 200 members are considered, which are famous trusses in the field of structural optimization. The results are compared with the previous results obtained through various existing metaheuristics. The algorithm was coded in Matlab on an Intel Core-i7 computer with 16 GB of RAM. Each problem was run 50 times and the results in the tables are in terms of minimum weight, mean, standard deviation (SD) and number of iterations (NI). The main input data of the problems are presented in Table 1.

Table 1. Input data for each problem

Problem	**Modules of elasticity E (N/m^2)**	**Weight density ρ (kg/m^3)**	**Size variables (cm^2)**	**Frequency constraints (Hz)**
72-bar spatial truss	$6{,}98 \times 10^{10}$	2770	$0{,}645 \leq A_i \leq 30$	$f_1 = 4$ $f_3 \geq 6$

120-bar spatial truss	$2{,}1\text{x}10^{11}$	7971,81	$1 \le A_i \le 129{,}3$	$f_1 \ge 9$ $f_2 \ge 11$
72-bar planar truss	$2{,}1\text{x}10^{11}$	7860	$0{,}1 \le A_i \le 30$	$f_1 \ge 5$ $f_2 \ge 10$ $f_3 \ge 15$

4.1 72-bar spatial truss

The first example aims to optimize a 72-bar spatial truss whose representation in geometry and elements is represented in Figure 2. This problem is considered as a large-scale dimensional problem (Tejani, Savsani, Patel, & Mirjalili, 2018). Table 1 presents the design parameters for this problem. The truss elements are categorized into sixteen groups of members considering structural symmetry (Kaveh & Ilchi Ghazaan, 2017). A lumped mass of 2770 kg is added to all upper nodes (nodes 1 to 4) as shown in Figure 2.

Table 2 provides the optimal results obtained with SAwPSO and different algorithms. The optimal design achieved by SAwPSO has a weight of 324.97 kg which is less than the results reported by CSS-BBBC (327.51 kg), DPSO (327.65 kg), SBO (327.55 kg), VPS (327.65 kg) and ISOS (325.01 kg) and 0.72 kg heavier than those reported by ReDE and AHEFA. In terms of convergence speed, it is observed that the NI used by the DPSO and VPS algorithms is higher when compared to SAwPSO (20000 NI for DPSO, 30000 for VPS 15000 for SAwPSO). Finally, it can be seen from the results that SAwPSO obtains better results in mean weight and SD than other metaheuristics. Figure 3 presents the box plot for this problem.

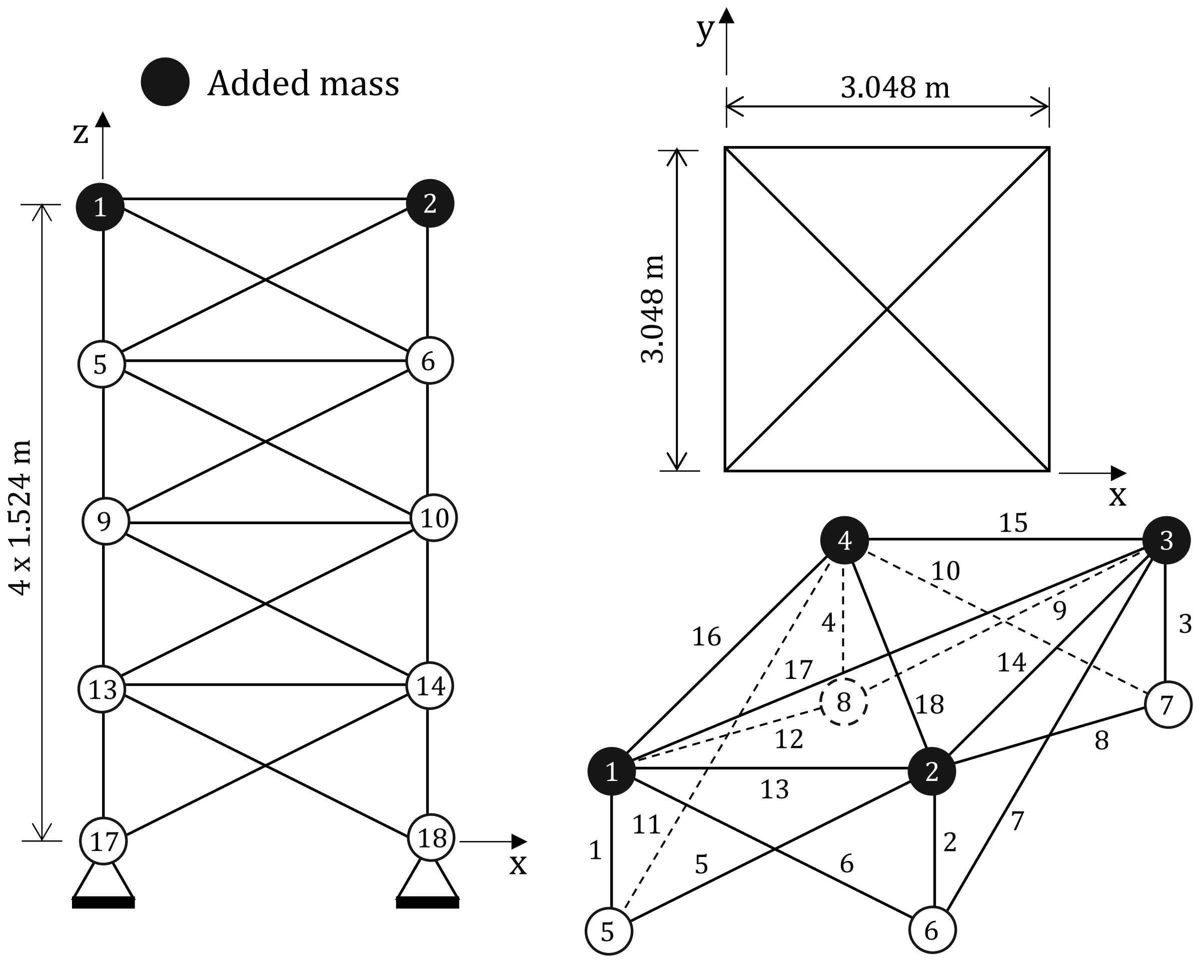

Figure 2. Schematic of the 72-bar spatial truss.

Table 2. Optimal design parameters for the 72-bar spatial truss by different algorithms

Variables (cm^2)	**Kaveh & Zolghadr (2012)**	**Kaveh & Zolghadr (2014)**	**Farshchin et al. (2016)**	**Kaveh & Ilchi Ghazaan (2017)**	**Ho-Huu et al. (2018)**	**Tejani et al. (2018)**	**Lieu et al. (2018)**	**SAwPSO**
	CSS-BBBC	**DPSO**	**SOB**	**VPS**	**ReDE**	**ISOS**	**AHEFA**	
A_1-A_4	2.854	3.5498	3.4917	3.5017	3.5327	3.3563	3.5612	3.5097
A_5-A_{12}	8.301	7.8356	7.9414	7.9340	7.8303	7.8726	7.8736	7.9228
A_{13}-A_{16}	0.645	0.6450	0.6450	0.6450	0.6453	0.6450	0.6450	0.6450
A_{17}-A_{18}	0.645	0.6450	0.6450	0.6450	0.6459	0.6450	0.6451	0.6462
A_{19}-A_{22}	8.202	8.1183	8.1154	8.0215	8.0029	8.5798	7.9710	8.0131
A_{23}-A_{30}	7.043	8.1338	8.0533	7.9826	7.9135	7.6566	7.8928	7.8949
A_{31}-A_{34}	0.645	0.6450	0.6450	0.6450	0.6451	0.7417	0.6450	0.6450
A_{35}-A_{36}	0.645	0.6450	0.6450	0.6450	0.6451	0.6450	0.6451	0.6457
A_{37}-A_{40}	16.328	12.6231	12.8569	12.8175	12.7626	13.0864	12.5404	12.7528
A_{41}-A_{48}	8.299	8.0971	8.0425	8.1129	7.9657	8.0764	7.9639	8.0182

A_{49}-A_{52}	0.645	0.6450	0.6451	0.6450	0.6452	0.6450	0.6459	0.6450
A_{53}-A_{54}	0.645	0.6450	0.6450	0.6450	0.6450	0.6937	0.6462	0.6455
A_{55}-A_{58}	15.048	17.3908	17.2136	17.3362	16.9041	16.2517	17.1323	16.8716
A_{59}-A_{66}	8.268	8.0634	8.0804	8.1010	8.0434	8.1703	8.0216	8.0260
A_{67}-A_{70}	0.645	0.6450	0.6450	0.6450	0.6451	0.6450	0.6450	0.6450
A_{71}-A_{72}	0.645	0.6450	0.6450	0.6450	0.6473	0.6450	0.6451	0.6450
Weight (kg)	327.51	327.65	327.55	327.65	324.25	325.01	324.24	324.97
f_1 (Hz)	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
f_3 (Hz)	6.0040	6.0000	6.0000	6.0000	6.0001	6.0008	6.0000	6.0000
Mean (kg)	–	327.76	327.68	327.67	324.32	329.47	324.41	325.38
SD (kg)	–	0.06	0.07	0.02	0.05	2.66	0.24	0.40
NI	–	20000	15000	30000	10840	4000	8860	15000

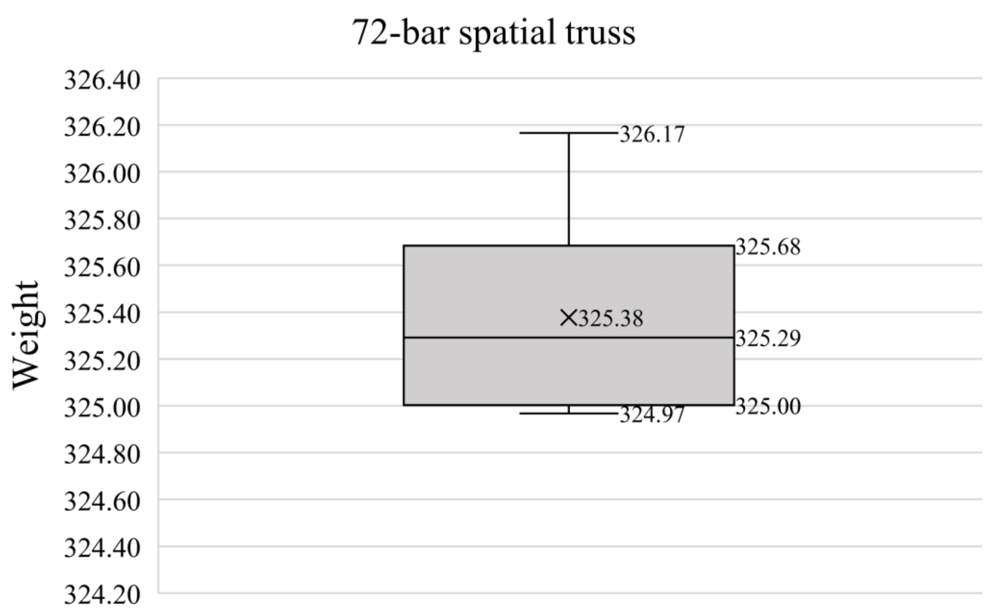

Figure 3. Box plot for the 72-bar spatial truss.

4.2 120-bar spatial truss

Figure 4 shows the 120-bar dome. Members are categorized into seven groups because of symmetry. Design considerations are tabulated in Table 1. Free nodes have the following concentrated masses: 3000 kg at node 1, 500 kg at nodes 2 to 13 and 100 kg at other free nodes.

Table 3 presents the results obtained using the proposed algorithm and other metaheuristics. It can be seen that the best SAwPSO design gives weight benefits of 340.01 kg, 183.15 kg, 182.63 kg, 181.41 kg and 2.73 kg compared to those obtained by the CSS-BBBC, DPSO , HALC-PSO, VPS and ISOS algorithms, respectively. On the other hand, the SAwPSO design is 0.19 kg slightly heavier when compared to the AHEFA algorithms respectively. Regarding the speed of convergence, SAwPSO

(15000 NI) ranks third among the considered metaheuristics. Finally, it can be observed from the results that SAwPSO obtains better results in mean weight and SD than other metaheuristics, only being surpassed by the AHEFA algorithm. Figure 5 presents the box plot for this problem.

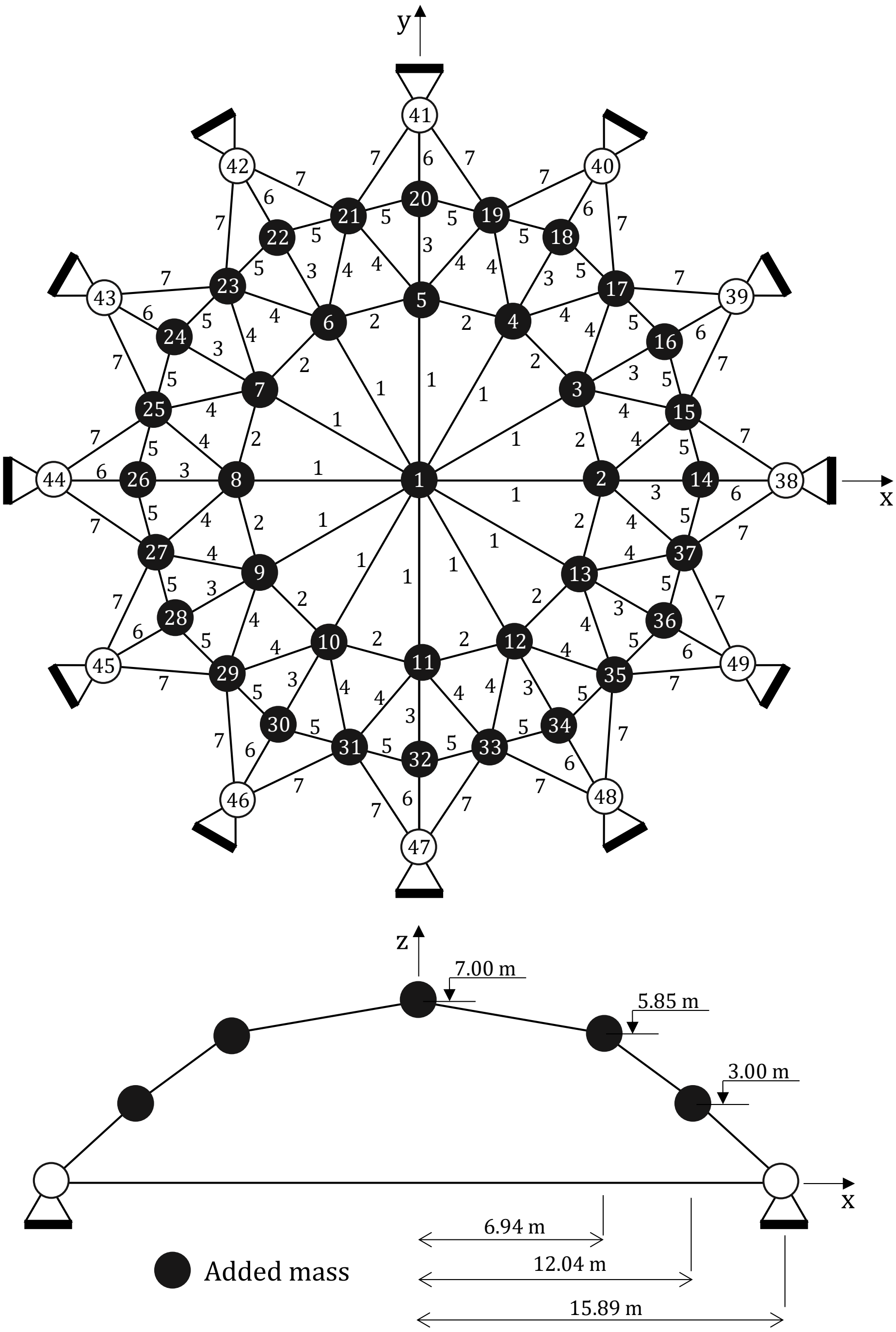

Figure 4. Schematic of the 120-bar spatial truss.

Table 3. Optimal design parameters for the 120-bar spatial truss by different algorithms

Variables (cm^2)	**Kaveh & Zolghadr (2014)**	**Kaveh & Ilchi Ghazaan (2015)**	**Kaveh & Ilchi Ghazaan (2017)**	**Tejani et al. (2018)**	**Lieu et al. (2018)**	**SAwPSO**
	DPSO	**HALC-PSO**	**VPS**	**ISOS**	**AHEFA**	
A_1	19.607	19.8905	19.6836	19.6662	19.5094	20.0425
A_2	41.290	40.4045	40.9581	39.8539	40.3867	39.4775
A_3	11.136	11.2057	11.3325	10.6127	10.6033	13.6425
A_4	21.025	21.3768	21.5387	21.2901	21.1168	20.4928
A_5	10.060	9.8669	9.8867	9.7911	9.8221	9.0488
A_6	12.758	12.7200	12.7116	11.7899	11.7735	15.2658
A_7	15.414	15.2236	14.9330	14.7437	14.8405	12.9846
Weight (kg)	8890.48	8889.96	8888.74	8710.06	8707.26	8707.45
f_1 (Hz)	9.0001	9.000	9.0000	9.0001	9.0000	9.0000
f_2 (Hz)	11.0007	11.000	11.0000	10.9998	11.0000	11.0000
Mean (kg)	8895.99	8900.39	8896.04	8728.56	8707.56	8710.96
SD (kg)	4.26	6.38	6.65	14.23	0.25	3.8
NI	6000	17000	30000	4000	3560	15000

Figure 5. Box plot for the 120-bar spatial truss.

4.3 200-bar planar truss

The third problem, illustrated in Figure 6, is considered a large-scale dimensional problem. Table 11 lists the material properties, frequency constraints, and limits of the problem variables. A lumped mass of 100 kg is added to all upper nodes (nodes 1 to 5). The bars are grouped into twenty-nine groups through the symmetry of the structure.

Table 4 lists the SAwPSO results compared to other metaheuristics. The best weight obtained by SAwPSO is 2157.58 kg, which is less than that given by CSS-BBBC (2298.61 kg), SOS (2180.32 kg), ISOS (2169.46 kg) and AHEFA (2160.74 kg), and slightly higher than HALC-PSO (2156.73 kg) and SBO (2156.51 kg). Regarding the speed of convergence, SAwPSO (15000 NI) is the fifth among the considered algorithms. From a statistical point of view, SAwPSO is more stable than SOS and ISOS with a lower DP (2.54 kg for SAwPSO, 83.59 kg for SOS and 43.48 kg for ISOS). Figure 7 presents the box plot for this problem.

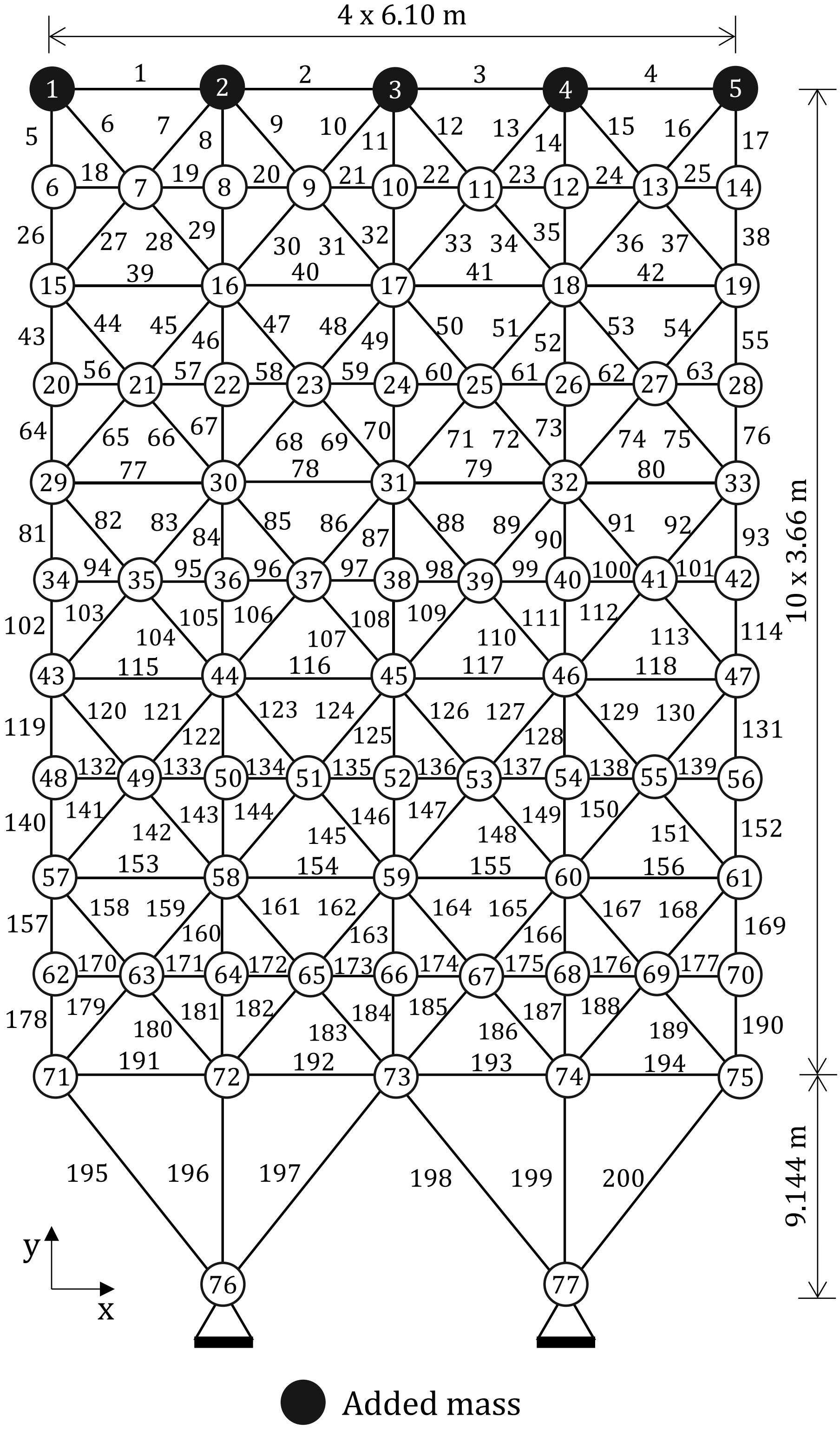

Figure 6. Schematic of the 200-bar planar truss.

Table 4. Optimal design parameters for the 200-bar planar truss by different algorithms

Variables (cm²)	Kaveh & Ilchi Ghazaan (2015)	Farshchin et al. (2016)	Tejani et al. (2016a)	Tejani et al. (2018)	Lieu et al. (2018)	SAwPSO
	HALC-PSO	SBO	SOS	ISOS	AHEFA	
A_1	0.3072	0.3040	0.4781	0.3072	0.2993	0.3072
A_2	0.4545	0.4478	0.4481	0.5075	0.4508	0.4680
A_3	0.1000	0.1000	0.1049	0.1001	0.1001	0.1000
A_4	0.1000	0.1000	0.1045	0.1000	0.1000	0.1000
A_5	0.5080	0.5075	0.4875	0.5893	0.5123	0.5483
A_6	0.8276	0.8219	0.9353	0.8328	0.8205	0.8173
A_7	0.1023	0.1003	0.1200	0.1431	0.1011	0.1006
A_8	1.4357	1.4240	1.3236	1.3600	1.4156	1.4832
A_9	0.1007	0.1001	0.1015	0.1039	0.1000	0.1000
A_{10}	1.5528	1.5929	1.4827	1.5114	1.5742	1.5285
A_{11}	1.1529	1.1597	1.1384	1.3568	1.1597	1.1262
A_{12}	0.1522	0.1275	0.1020	0.1024	0.1338	0.1378
A_{13}	2.9564	2.9765	2.9943	2.9024	2.9672	2.6452
A_{14}	0.1003	0.1001	0.1562	0.1000	0.1000	0.1004
A_{15}	3.2242	3.2456	3.4330	3.4120	3.2722	3.1630
A_{16}	1.5839	1.5818	1.6816	1.4819	1.5762	1.5654
A_{17}	0.2818	0.2566	0.1026	0.2587	0.2562	0.3406
A_{18}	5.0696	5.1118	5.0739	4.8291	5.0956	4.6363
A_{19}	0.1033	0.1001	0.1068	0.1499	0.1001	0.1087
A_{20}	5.4657	5.4337	6.0176	5.5090	5.4546	5.2540
A_{21}	2.0975	2.1016	2.0340	2.2221	2.0933	2.0678
A_{22}	0.6598	0.6794	0.6595	0.6113	0.6737	0.5970
A_{23}	7.6585	7.6581	6.9003	7.3398	7.6498	7.2983
A_{24}	0.1444	0.1006	0.2020	0.1559	0.1178	0.1554
A_{25}	8.0520	7.9468	6.8356	8.6301	8.0682	7.9421
A_{26}	2.7889	2.7835	2.6644	2.8245	2.8025	2.6696
A_{27}	10.4770	10.5277	12.1430	10.8563	10.5040	11.0528
A_{28}	21.3257	21.3027	22.2484	20.9142	21.2935	21.6370
A_{29}	10.5111	10.6207	8.9378	10.5305	10.7410	11.0438
Weigth (kg)	2156.73	2156.51	2180.32	2169.46	2160.74	2157.48
f_1 (Hz)	5.000	5.000	5.0001	5.0000	5.0000	5.0000
f_2 (Hz)	12.254	12.2141	13.4306	12.4477	12.1821	12.2691
f_3 (Hz)	15.044	15.0192	15.2645	15.2332	15.0160	15.0023
Mean (kg)	2157.14	2156.79	2303.30	2244.64	2161.04	2162.77

SD (kg)	0.24	0.21	83.59	43.48	0.18	2.54
NI	13000	23000	10000	10000	11300	15000

Figure 7. Box plot for the 200-bar planar truss.

5. CONCLUSION

In this work, the performance of the Simulated Annealing with Particle Swarm (SAwPSO) algorithm for optimal design of truss structures with multiple frequency constraints algorithm in was evaluated. Three numerical examples (72-bar spatial truss, 120-bar spatial truss and 200-bar planar truss) are investigated to prove the efficiency of the SAwPSO. The results indicate that the algorithm can obtain an acceptable solutions in terms of minimum weight, mean, standard deviation and number of iterations.

REFERENCES

Bellagamba, L., & Yang, T. Y. (1981). Minimum-mass truss structures with constraints on fundamental natural frequency. AIAA Journal, 19(11), 1452–1458. https://doi.org/10.2514/3.7875

Farshchin, M., Camp, C. V., & Maniat, M. (2016). Optimal design of truss structures for size and shape with frequency constraints using a collaborative optimization strategy. Expert Systems with Applications, 66, 203–218. https://doi.org/10.1016/j.eswa.2016.09.012

Gomes, H. M. (2011). Truss optimization with dynamic constraints using a particle swarm algorithm. Expert Systems with Applications, 38(1), 957–968. https://doi.org/10.1016/j.eswa.2010.07.086

Gonçalves, M. S., Lopez, R. H., & Miguel, L. F. F. (2015). Search group algorithm: A new metaheuristic method for the optimization of truss structures. Computers & Structures, 153, 165–184. https://doi.org/10.1016/j.compstruc.2015.03.003

Grandhi, R. V., & Venkayya, V. B. (1988). Structural optimization with frequency constraints. AIAA Journal, 26(7), 858–866. https://doi.org/10.2514/3.9979

Ho-Huu, V., Nguyen-Thoi, T., Truong-Khac, T., Le-Anh, L., & Vo-Duy, T. (2018). An improved differential evolution based on roulette wheel selection for shape and size optimization of truss structures with frequency constraints. Neural Computing and Applications, 29(1), 167–185. https://doi.org/10.1007/s00521-016-2426-1

Kaveh, A., & Ilchi Ghazaan, M. (2015). Hybridized optimization algorithms for design of trusses with multiple natural frequency constraints. Advances in Engineering Software, 79, 137–147. https://doi.org/10.1016/j.advengsoft.2014.10.001

Kaveh, A., & Ilchi Ghazaan, M. (2017). Vibrating particles system algorithm for truss optimization with multiple natural frequency constraints. Acta Mechanica, 228(1), 307–322. https://doi.org/10.1007/s00707-016-1725-z

Kaveh, A., & Zolghadr, A. (2011). Shape and size optimization of truss structures with frequency constraints using enhanced charged system search algorithm. Asian J. Civil Eng., 12(4), 487–509.

Kaveh, A., & Zolghadr, A. (2012). Truss optimization with natural frequency constraints using a hybridized CSS-BBBC algorithm with trap recognition capability. Computers and Structures, 102–103, 14–27. https://doi.org/10.1016/j.compstruc.2012.03.016

Kaveh, A., & Zolghadr, A. (2014). Democratic PSO for truss layout and size optimization with frequency constraints. Computers and Structures, 130, 10–21. https://doi.org/10.1016/j.compstruc.2013.09.002

Lieu, Q. X., Do, D. T. T., & Lee, J. (2018). An adaptive hybrid evolutionary firefly algorithm for shape and size optimization of truss structures with frequency constraints. Computers & Structures, 195, 99–112. https://doi.org/10.1016/j.compstruc.2017.06.016

Lin, J. H., Che, W. Y., & Yu, Y. S. (1982). Structural optimization on geometrical configuration and element sizing with statical and dynamical constraints. Computers & Structures, 15(5), 507–515. https://doi.org/10.1016/0045-7949(82)90002-5

Miguel, L. F. F., & Fadel Miguel, L. F. (2012). Shape and size optimization of truss structures considering dynamic constraints through modern metaheuristic algorithms. Expert Systems with Applications, 39(10), 9458–9467. https://doi.org/10.1016/j.eswa.2012.02.113

Talbi, E. G. (2009). Metaheuristics: From Design to Implementation. Metaheuristics: From Design to Implementation. John Wiley and Sons. https://doi.org/10.1002/9780470496916

Tejani, G. G., Savsani, V. J., & Patel, V. K. (2016). Adaptive symbiotic organisms search (SOS) algorithm for structural design optimization. Journal of Computational Design and Engineering, 3(3), 226–249. https://doi.org/10.1016/j.jcde.2016.02.003

Tejani, G. G., Savsani, V. J., Patel, V. K., & Mirjalili, S. (2018). Truss optimization with natural frequency bounds using improved symbiotic organisms search. Knowledge-Based Systems, 143, 162–178. https://doi.org/10.1016/j.knosys.2017.12.012